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Non-commutative $U(1)$ Super-Yang–Mills Theory: Perturbative Self-Energy Corrections

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Abstract. The quantization of the non-commutative $\mathcal{N} = 1$, $U(1)$ super-Yang–Mills action is performed. We calculate the one-loop corrections to the self-energy of the super vector field. Although the power-counting theorem predicts quadratic ultraviolet and infrared divergences, there are actually only logarithmic UV and IR divergences unless one chooses the Wess–Zumino gauge, for which the divergences are indeed quadratic. This could indicate that UV/IR mixing might be unphysical.

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Dedicated to Olivier Piguet on the occasion of his 60th birthday

1 Introduction

We know that the concept of space-time as a differentiable manifold cannot be reasonably applied to extremely short distances [?]. Simple heuristic arguments show that it is impossible to locate a particle with arbitrarily small uncertainty [?]. An interesting concept in order to replace standard differential geometry is *non-commutative geometry* pioneered by Connes [?, ?]. Non-commutative geometry can be regarded as an extension of the principles of quantum mechanics to geometry itself: space-time coordinates become non-commutative operators.

The general strategy in non-commutative geometry is to generalize the mathematical structures encountered in ordinary physics. Standard quantum field theories deal with problems of interactions at short distances. Quantum field theory (QFT) on spaces with different short-distance structure may therefore show interesting features. Since singularities in standard QFT are a consequence of point-like interactions, there has been hope that 'smearing out the points' [?] avoids these UV divergences. However, it was first noticed by Filk [?] that divergences are not avoided on non-commutative $\mathbb{R}^4$. This raised the question of whether the QFT is renormalizable, or not. On the one-loop level this was affirmed for Yang–Mills theory on non-commutative $\mathbb{R}^4$ [?] and the non-commutative 4-torus [?] as well as for supersymmetric Yang–Mills theory in (2+1) dimensions, with space being the non-commutative 2-torus [?]. QED on non-commutative $\mathbb{R}^4$ was treated in [?] and BF–Yang–Mills theory in [?]. The Chern–Simons model on non-commutative space was treated in [?], see also [?], and the Wess–Zumino model in [?].

Concerning supersymmetry, also a deformation of the anticommutator of the fermionic superspace coordinates was considered [?], but this deformation is not compatible with supertranslations and chiral fields. A superspace formulation (at the classical level) of the Wess–Zumino model and of super-Yang–Mills theory was given in [?]. Non-commutative $\mathcal{N} = 1, 2$ super-Yang–Mills theories were studied by Zanon in [?], using the background field method, with the result that at one loop there are only logarithmic divergences in the self-energy. This is remarkable because the power-counting theorem predicts *quadratic divergences* for $\mathcal{N} = 1$ super-Yang–Mills theory, which would lead, according to the power-counting analysis of non-commutative field theories by Chepelev and Roiban [?], to non-renormalizability on non-commutative space-time. The lowering of the degree of divergence from quadratic to logarithmic seems to be governed by non-renormalization theorems, see [?].

In this paper we reinvestigate the question of UV/IR mixing in non-commutative $\mathcal{N} = 1$ super-Yang–Mills theory. We work in the non-commutative superfield formalism [?], which allows us to easily switch between a general superfield and one in the Wess–Zumino gauge. It turns out that the one-loop self-energy of the superfield is *quadratically* IR-divergent in the Wess–Zumino gauge and *logarithmically* IR-divergent for a general superfield. UV divergences are multiplicatively renormalizable in both cases. Assuming that this behaviour continues to all orders, non-commutative $\mathcal{N} = 1$ super-Yang–Mills theory would be renormalizable, according to [?], when using general superfields (provided that commutants-type divergences are absent), and non-renormalizable when